Lecture 3. Functioning and development of the system

The purpose of the lecture: to discuss an introduction to the fundamentals of systems activity - functioning and development, self-development, the necessary mathematical apparatus for their consideration - the algebra of relations. Consider the basic concepts concerning the behavior of systems - the functioning and development (evolution), as well as the self-development of systems, necessary for their study of the concept of the theory of relations and order.

Lecture plan:
Introduction
1 System operation
2 Developing systems
3 Algebraic structure
Conclusion

Keywords: activity, evolution, functioning, development, infrastructure, informatization, energy, interpretation, program, background mode, information, attitude, developing system, structure, stable system, segment, self-developing, index, HDI, development, INDEX, queue, system flexibility, flexibility, system, regulation, trajectory, correction, set of states, space, system analysis, sets, subset, generality quantifier, existence quantifier, composition, work, reflexivity, antisymmetry, partial ordering, ordered, order, place, definition, lattice, tuple, arbitrary, closed system, open system, equivalence class, partition of set, remainder, equivalent system, invariant, domain, deployment, cost, costs, correspondence, operations, selection, algebra, algebraic, signature, carrier, unary operation , groupoid, semigroup, axiom, alphabet, group, binary operation i, inverse, isomorphism, ring, multiplication, addition, field.

Contents of the lecture:

Introduction

The activity (work) of the system can occur in two main modes: development (evolution) and functioning.

1 System operation

The activity (work) of the system can occur in two main modes: development (evolution) and functioning.

Functioning is an activity, the operation of a system without changing the (main) goal of the system. This is a manifestation of the function of the system in time.

Development is the activity of a system with a change in the goal of the system.

During the functioning of the system, there is clearly no qualitative change in the system infrastructure; with the development of the system, its infrastructure changes qualitatively.

Development is the struggle between organization and disorganization in the system; it is associated with the accumulation and complication of information and its organization.

Example. Informatization of the country at its highest stage - the full use of various knowledge bases, expert systems, cognitive methods and tools, modeling, communication tools, communication networks, providing information and, therefore, any security, etc .; it is a revolutionary change, the development of society. Computerization of society, region, organization without posing new urgent problems, i.e. "hanging computers on old methods and technologies of information processing" is a functioning, not a development. The decline of moral and ethical values in society, the loss of purpose in life can also lead to the "functioning" of not only individuals, but also social strata.

Any actualization of information is associated with the actualization of matter, energy and vice versa.

Example. Chemical development, chemical reactions, the energy of these reactions in human organisms lead to biological growth, movement, accumulation of biological energy; this energy is the basis of information development, information energy; the latter determines the energy of social movement and organization in society.

Example. Classically, it is believed that in the process of photosynthesis oxygen is released and carbon dioxide is absorbed (in plants, algae and some microorganisms) and at the same time, under the influence of light, carbon dioxide is released and oxygen is absorbed - respiration (or, more precisely, photorespiration) occurs. The bioenergetic equation of photosynthesis and respiration of plants (organisms) has the form

Figure 3.1 The bioenergetic equation of photosynthesis and respiration of plants (organisms)

The bioenergy informational version of this formula can have the form

Figure 3.2. The bioenergy informational version of this formula

This interpretation not only takes into account, but also helps to better understand the bioenergy informational development of the system and the complex information processes occurring in a biological system with energy flows.

Example. With high illumination and the presence of oxygen in the plant, an internal mechanism for absorbing carbon dioxide is triggered (that is, control is transferred to the "Absorption of carbon dioxide" program), which after starting can also occur in the dark, leading to the absorption of carbon dioxide or a decrease in photosynthesis (program "Oxygen release" goes into "background mode"). The corresponding information on the subsystems of the "Plant" system is transmitted along the fibers of the plants.

If in a system quantitative changes in the characteristics of elements and their relationships lead to qualitative changes, then such systems are called developing systems. Developing systems have a number of distinctive aspects, for example, they can spontaneously change their state as a result of interaction with the environment (both deterministically and randomly). In developing systems, the quantitative growth of elements and subsystems, system connections leads to qualitative changes (systems, structures), and the viability (stability) of the system depends on changes in the connections between the elements (subsystems) of the system.

Example. The development of language as a system depends on the development and connections of the constituent elements - words, concepts, meaning, etc. The formula for Fibonacci numbers: $x_n = x_{n-1} + x_{n-2}$, $n > 2$, $x_1 = 1$, $x_2 = 1$ uniquely determines the developing system of numbers. If we consider the numbers: 1, 1, 2, 5, 29, ..., then it is easy to see that the initial segment is similar to the Fibonacci series, but this impression is deceiving. In fact, each member of the series (from the third) is obtained not by adding the previous two, but by adding their squares. Mathematically, this law is written in a completely different form: $xn = (x_{n-1})^2 + (x_{n-2})^2$, $n = 3, 4, \ldots$. Thus, in the "numerical notation" of the series, in contrast to the analytical one, there was some instability, since specifying only the first four members of this series could lead to incorrect conclusions about the behavior of the system.

2 Developing systems

The main features of developing systems:

- ✓ *spontaneous change in the state of the system;*
- ✓ *counteraction (reaction) to the influence of the environment (other systems), leading to a change in the initial state of the environment;*
- ✓ *constant flow of resources (constant work on their cross-flow "environment-system"), directed against balancing their flow with the environment.*

If a developing system evolves at the expense of its own material, energy, information, human or organizational resources within the system itself, then such systems are called self-developing (self-developing). This is a form of system development - "the most desirable" (for the set goal).

Example. If the demand for skilled labor increases on the labor market, then there will be a desire to increase qualifications, education, which will lead to the emergence of new educational services, qualitatively new forms of advanced training, for example, distance learning. The development of a company, the emergence of a network of branches can lead to new organizational forms, in particular, to a computerized office, moreover, to the highest stage of development of an automated office - a virtual office or a virtual corporation. Lack of time for shopping, for example, among employed and computer literate young people with sufficient income ("yuppies"), influenced the emergence and development of Internet commerce.

To assess the development and development of the system, not only qualitative, but also quantitative assessments, as well as mixed assessments, are often used.

Example. In the UN system, to assess the socio-economic development of countries, the HDI (Human Development Index) index is used, which takes into account 4 main parameters, which vary from their minimum to their maximum values:

1. *life expectancy of the population (25-85 years);*
2. *adult illiteracy rate (0-100%);*
3. *average duration of schooling of the population (0-15 years);*
4. *annual per capita income ($ 200-40,000).*

This information is reduced to the total HDI value, according to which all countries are divided by the UN into highly developed, medium-developed and underdeveloped. Countries with developing (self-developing) economic, legal, political, social, educational institutions are characterized by a high level of HDI. In turn, a change in the HDI level (the parameters on which it depends) affects the self-development of these institutions, primarily economic ones, in particular, the self-regulation of supply and demand, producer-consumer relations, goods and costs, training and training costs. The HDI level, on the contrary, can also lead to the transition of a country from one category (development according to this criterion) to another, in particular, if in 1994 Russia was in 34th place in the world (out of 200 countries), then in 1996 it was already 57th place; this leads to changes in the relationship with the environment (in this case, in politics).

System flexibility will be understood as the ability to structurally adapt the system in response to environmental influences.

Example. The flexibility of the economic system is the ability to structurally adapt to changing socio-economic conditions, the ability to regulate, to changes in economic characteristics and conditions.

The trajectory of the system is determined by its structure, elements, environment. For simple systems (we will understand such systems as systems not free in the choice of behavior), the trajectory can be changed only by changing the elements, structure, and environment. For complex (complex - below they are discussed in more detail) systems, a change in trajectory can occur for other reasons.

Regulation (of a system, system behavior, system trajectory) is understood as the correction of control parameters by observing the trajectory of the system's behavior in order to return the system to the desired state, to the desired trajectory of behavior. The trajectory of a system is understood as a sequence of states accepted during the functioning of the system, which are considered as some points in the set of states of the system. For physical, biological and other systems, this is phase space.

3 Algebraic structure

To formalize facts in systems analysis (as in mathematics, computer science and other sciences), the concepts of "relation" and "algebraic structure" are used.

The relation r defined over the elements of a given set X is a certain rule according to which each element x \ in X is associated with another element (or other elements) y \ in X. The relation r is called an n-ary relation if it connects n different elements of X. The set of pairs (x, y), which are in a binary (binary) relation to each other, is a subset of the Cartesian set X×Y. The ratio of r elements x \ in X, y \ in Y is denoted as, $x \xrightarrow{r} y$ or r(X, Y).

Example. Consider a classical computer circuit of devices: 1 - input, 2 - logic-arithmetic, 3 - control, 4 - memory, 5 - output. We define the relation "information exchange" as follows: device i is in relation r with device j if information is received from device i to device j. Then this relation can be determined by the matrix R of relations (the presence of r at the intersection of row i and column j indicates that device i is in this relation with device j, and the presence of Ø indicates the absence of this relation between them):

$$R = \begin{vmatrix} \emptyset & r & r & r & \emptyset \\ \emptyset & \emptyset & r & r & r \\ r & r & \emptyset & r & r \\ \emptyset & r & r & \emptyset & r \\ \emptyset & \emptyset & r & \emptyset & \emptyset \end{vmatrix}$$

The relation given by the phrase "for each x \ in X" is denoted $\forall x \in X$ and is called the generality quantifier, and the relation "exists x \ in X" is denoted $\exists x \in X$ and is called the existential quantifier. The fact that the elements $x \in X$ are connected, distinguished by some relation r, is denoted as X = {x: r} or X = {x | r}.

Composition (product) $r = r_1 \times r_2$. relations r_1 and r_2, given over the same set X, is the third relation r determined by the rule:

$$x \xrightarrow{r} y \Leftrightarrow \left((\exists z \in X): \left(x \xrightarrow{r} z\right), \left(z \xrightarrow{r} y\right)\right)$$

The relation r is called the relation 1) identity; 2) reflective; 3) mpanzitive; 4) symmetrical; 5) inverse to the relation s if, respectively, the conditions

1. $x \xrightarrow{r} y \Leftrightarrow (x = y)$
2. $(\forall x \in X): x \xrightarrow{r} x$
3. $x \xrightarrow{r} y,\ y \xrightarrow{r} z \ \Rightarrow\ x \xrightarrow{r} z$
4. $x \xrightarrow{r} y \ \Rightarrow\ y \xrightarrow{r} x$
5. $x \xrightarrow{r} y \ \Leftrightarrow\ y \xrightarrow{r} x$

Example. The binary relation of equality of numbers "=" is reflexive (since x = x), symmetric (since x = y → y = x), transitive (since x = y, y = z → x = z). The binary relation "have a common divisor" is reflexive, symmetric, transitive (check). Binary relation of nesting of sets "⊆" - reflexive, antisymmetric, transitive (check).

A system X partially ordered with respect to r is a system for which (i.e., for any elements of which) the relation r(X) is given, which is transitive, asymmetric, reflexive.

A system ordered with respect to r(X) is a system X such that $\forall x, y \in X$, either, $x \xrightarrow{r} y$ or $y \xrightarrow{r} x$.

A system with a partial ordering relation given on it (on the set of elements defining it) is called a system with order, and a system with a given ordering relation is called a system with full order.

Example. Let N be the set of natural numbers. The relation r(x, y): "x is multiple of y" defined on N, as it is easy to check, is a partial ordering. The ratio r(x, y): "x ≤ y" defined on the set of real numbers R is a partial order and total order ratio. The relation r (x, y): "x <y" defined on R is not a full order relation (not reflexive). The nesting relation of sets "$X \subseteq Y$" is a partial ordering relation of sets defined on the set of all sets, but it is not a full ordering relation (not every two sets are included in one direction or another).

Now we can give a formalized definition of the concept of structure.

A structure defined over a set (or on a set) X is a certain relation over X of the ordering type. A more formal, mathematical definition: structure (lattice) is a partially ordered set X for which any two-element subset {x, y} of X has the largest or smallest element (supremum or infinum).

Thus, the system can be understood as an integral complex (tuple) of objects S = <A, R>, A = {a}, R = {r}, where r is a relation over A, A is an arbitrary set of elements. Such a system is called a closed system. In closed systems, an important characteristic of the functioning of the system is the internal structure of the system. Closed systems are an abstract product, a product of thinking, logical construction. They are limited ("closed") by the level of their theoretical consideration.

If Y is a set of elements of the external (in relation to A) environment C, and relations r over C are defined in C, then the tuple S = <A, Y, R> sets, defines an open system. In open systems, an important characteristic of functioning is the exchange of resources (of one or more types) with other systems, with the environment, as well as the nature of this exchange.

A transitive, reflexive, symmetric relation is called an equivalence relation. The equivalence relation r(X) divides the set of systems X into classes or classes of equivalence - nonempty and disjoint sets of systems, each of which, together with any of its elements, also contains all elements of X that are equivalent to it with respect to r(X), and does not contain other x from X.

Theorem. Two equivalence classes over the same set do not intersect. If two elements x, y from X are not related by the equivalence relation r(x, y) defined on X, then the equivalence classes for these elements do not intersect. If an equivalence relation r(x, y), x, y from X is given on a set X, and X_x, X_y are equivalence classes with respect to x, y, respectively, then $X_x = X_y$.

Example. The relation between x, y, expressed by the equality x = y + ka, x, y, k, a from Z, is called the comparison relation between x and y modulo a and is written as x = y (mod a). This relation is an equivalence relation:

1. x = x (mod a), k = 0 (reflexivity);
2. x = y (mod a) → x = y + ka → y = x + (- k) a → y = x (mod a) (symmetry);
3. x = y (mod a), y = z (mod a) → x = y + ka, y = z + ma => x = z + (k + m) a → x = z (mod a) (transitivity).

The set of integers Z is divided by this relation into k classes:

$X_0 = \{x: x = ka, k, a \in Z\}$,

$X_1 = \{x: x = 1 + ka, k, a \in Z\}$,

$X_2 = \{x: x = 2 + ka, k, a \in Z\}$,

……………………………..

$X_{k-1} = \{x: x = k-1 + ka, k, a \in Z\}$.

In particular, for k = 2, the set Z is partitioned into the set X_0 - even and the set X_1 - odd numbers; for k = 3, the set Z is divided into classes X_0 - multiples of 3, X_1 - giving remainder 1 when divided by 3, X_2 - giving remainder 2 when divided by 3.

We will call two systems equivalent if they have the same goals, constituent elements, structure. A relationship (strictly speaking, equivalence) can be established between such systems in some constructive way.

You can also talk about a "weakened" type of equivalence - equivalence in terms of the goal (elements, structure).

Let two equivalent systems X and Y be given and system X has a structure (or property, value) I. If this implies that system Y also has this structure (or property, value) I, then I is called an invariant of systems X and Y. One can talk about the invariant content of two or more systems or about the invariant immersion of one system into another. The invariance of two or more systems presupposes the presence of such an invariant.

Example. If we consider the process of cognition in any subject area, cognition of any system, then the global invariant of this process is its spiral nature. Consequently, the spiral of cognition is an invariant of any cognitive process, independent of external conditions and states (although the parameters of the spiral and its deployment, for example, the speed and steepness of deployment, depend on these conditions). Price is an invariant of economic relations, economic system; it can determine both money and cost and costs. The concept "system" is an invariant of all areas of knowledge.

The correspondence S is a binary relation r over the set X×Y:

$$S = \left\{(x, y): \left(x \overset{r}{\to} y\right), (x, y) \in X \times Y\right\}.$$

The inverse correspondence to r is a correspondence $S^{-1} \subseteq Y \times X$ of the form

$$S^{-1} = \left\{(y, x): \left(x \overset{r}{\to} y\right), (x, y) \in X \times Y\right\}$$

Relationships are often used in organizing and formalizing systems. At the same time, the following basic operations are introduced for them (over them):

1. *the union of two relations $r_1(x_1, x_2, ..., x_n)$, $r_2(x_1, x_2, ..., x_n)$, given over the set X, is the third relation $r_3(X) = r_1 \cup r_2$ obtained as the set-theoretic union of all elements of X for which r_1 or r_2 is valid;*
2. *intersection - $r_3(X) = r_1 \cap r_2$ - set-theoretic intersection of all elements from X for which r_1 and r_2 are valid;*
3. *the projection of the ratio r1(X) of dimension k, i.e. relations $r_1 = r_1(x_1, x_2, ..., x_k)$ connecting elements $x_1, x_2, ..., x_k$ from X (these may not be the first k elements), is the ratio r_2 of dimension m <k, i.e. it uses some of the arguments (parameters) of the original relation;*
4. *the difference of two ratios $r_1(x_1, x_2, ..., x_k)$, $r_2(x_1, x_2, ..., x_k)$ is the ratio $r_3 = r_1 - r_2$ consisting of all those elements of X for which the relation r_1 is valid, but the relation r_2 is not true;*
5. *the Cartesian product of two relations $r_1(x_1, x_2, ..., x_n)$ and $r_2(x_{n+1}, x_{n+2}, ..., x_{n+m})$ is the ratio $r_3 = r_1 \times r_2$ composed of all possible combinations of all elements of X for which the relations r_1, r_2 are valid; the first n components of the relation r_3 form elements for which the relation r_1 is valid, and for the last m elements the relation r_2 is valid;*
6. *selection (sampling) according to the criterion of q components belonging to the relation r; criterion q is some predicate.*

Relational algebras are often called relational algebras.

In connection with the use of the intuitively known concept of "algebra", let us clarify this structure, since it is often used as the main apparatus of the most formalized description of systems. Algebra is the most adequate mathematical apparatus for describing actions with letters, therefore, algebraic methods are best suited for describing and formalizing various information systems.

An algebra A = <X, f> is a collection of certain elements of X, with certain operations f (often defined by their similarity to the operations of addition and multiplication of numbers) that satisfy certain properties - the axioms of algebra.

An operation f is called n-ary if it concatenates n operands (objects - participants of this operation).

The collection F = {f} of operations of the algebra A is called its signature, and the collection of elements X = {x} is called the support of the algebra.

A Boolean algebra is an algebra with two two-place operations introduced into it, which are named, by analogy with the arithmetic of numbers, addition and multiplication, and one single operation called a prime operation or inversion, and these operations satisfy the axioms (laws) of the Boolean algebra:

1. *commutativity* - $x + y = y + x$, $xy = yx$;
2. *associativity* - $(x + y) + z = x + (y + z)$, $(xy) z = x (yz)$;
3. *idempotency* - $x + x = x$, $xx = x$;
4. *distributivity* - $(x + y) z = xz + yz$, $xy + z = (x + z) (y + z)$;
5. *involution (double inversion) -;*
6. *absorption* - $x (x + y) = x$, $x + xy = x$;
7. *de Morgan* - $x + y = xy$, $xy = x + y$
8. *neutrality:* $x (y + y) = x$, $x + yy = x$.
9. *existence of two special elements (called "unit -1" and "zero-0"), and* $0 = 1$, $1 = 0$, $x + x = 1$, $xx = 0$.

A groupoid is an algebra A = <X, f> with one two-place operation f.

Semigroup - groupoid, in the system of axioms of which there is the axiom of associativity. Therefore, it is called an associative groupoid.

Example. Let $X = \{x_1, x_2, ..., x_n\}$ be some alphabet. Then it forms a semigroup with respect to the operation of concatenation of words from S(X). In such (called free) semigroups, one of the most important algebraic problems of computer science in semigroups is considered - the word identity problem: to indicate a constructive process for establishing the coincidence of two words from the semigroup S(X). This problem is algorithmically unsolvable and occurs, for example, in the development of processor architecture.

Group is a semigroup with unit (with element e: ea = ae = a), in which the binary operation f is uniquely invertible, i.e. on this set (on its support), equations of the form xfa = b, afx = b are uniquely solvable.

Example. Let $X = \{x_1, x_2, ..., x_n\}$ be some free semigroup. To each of x_i, i = 1, 2, ..., n, we associate its inverse element x_i^{-1}, and set one equal to the empty word Ø. Then X forms a (free) group if, as a criterion for the solvability of the equations, we choose the relations: $x_i x_i^{-1} = Ø$, $x_i^{-1} x_i = Ø$. One of the most important algebraic problems of computer science in groups is the problem of isomorphism (transformation with preservation of a group operation) of two groups: to indicate a constructive process of establishing such a transformation from one group to another. This problem arises when processing information, converting one information system to another while preserving information.

A ring is an algebra with two binary operations: according to one of them (multiplication) it is a groupoid, and according to the other (addition) it is a group with the commutativity axiom (abelian group), and these operations are related to each other by the distributivity axioms.

A field is a ring in which all nonzero elements form an abelian group by one of the operations.

Example. The set of rational, real numbers, square matrices - form both fields and rings.

An isomorphism of two ordered (with respect to r) sets X and Y is such a one-to-one correspondence f: $X \to Y$, where the fact that $x_1, x_2 \in X$ are in relation r implies such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$ are in relation r and vice versa.

Isomorphism allows you to explore the invariant, general (systemic) in structures, transfer knowledge (information) from one structure to another, and build and strengthen interdisciplinary connections.

A property can exist as a structure regardless of the system, its carrier, and the system provides (through its structure) the ability (potency) to the property to interact with other systems (with other properties of systems) that have the same property.

Conclusion

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Control questions

See the manual on the organization of students' independent work.